Useful facts about lognormal distribution

Density of lognormal $LN(\mu, \sigma^2)$:

$$f_{LN(\mu,\sigma^2)}(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \tag{1}$$

CDF $LN(\mu, \sigma^2)$:

$$\int_0^z \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\ln x - \mu)^2)}{2\sigma^2}\right] dx = \Phi\left(\frac{\ln z - \mu}{\sigma}\right) \tag{2}$$

Expectation of lognormal $LN(m, \sigma^2)$:

$$\int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\ln x - \mu)^2)}{2\sigma^2}\right] dx = e^{\mu + \frac{\sigma^2}{2}} \tag{3}$$

Variance $LN(\mu, \sigma^2)$:

$$\int_0^\infty \left(x - e^{\mu+\frac{\sigma^2}{2}}\right)^2 f_{LN(\mu,\sigma^2)}\, dx = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right) = (\mathrm{E}x)^2\left(e^{\sigma^2} - 1\right) \tag{4}$$

Higher order moments:

$$\mu_k \equiv \int_0^\infty x^k f_{LN(\mu,\sigma^2)}(x)\, dx = \exp\left[k\mu + \frac{k^2}{2}\sigma^2\right] \tag{5}$$

Incomplete higher order moments:

$$\int_0^z x^k f_{LN(\mu,\sigma^2)}(x)\, dx = \mu_k CDF(z|\mu + k\sigma^2, \sigma^2) = \mu_k \Phi\left(\frac{\ln z - \mu - k\sigma^2}{\sigma}\right) \tag{6}$$

Lorenz curve at point $q \in [0, 1]$:

$$L(q) = \frac{1}{\mu_1}\int_0^z x f_{LN(\mu,\sigma^2)}(x)\, dx = \Phi\left(\frac{\ln z - \mu - \sigma^2}{\sigma}\right) \tag{7}$$

Estimate of σ from the Lorenz curve:

$$\sigma = \Phi^{-1}(q) - \Phi^{-1}(L(q)) \tag{8}$$

Gini coefficient:

$$Gini = 2\Phi(\sigma/\sqrt{2}) - 1 \tag{9}$$

References

[1] J. Aitchison, J. A. C. Brown. The Lognormal Distribution. Cambridge Univ. Press, 1963.