The Methods of the Quality of Life Assessment

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Abstract

This Master Thesis develops the concept of Quality of Life as an aggregate measure of people's well-being in a certain country, region, or social stratum. The historical approaches to the quality of life measurement and estimation are given in the intorduction and literature review. The United Nations approach to construction of a numerical measure of human development (human development index) is presented. A simple regression model and ranking according to the principal components of the set of economic and social indicators are proposed as ways to quantify the quality of life given its expert estimates as a training sample. This methodology was found to be an efficient one as indicated by sufficient statistical significance of both models. The success of the model served as a basis for application to rank the regions of Russian Federation according to the first principal component of the set of statistical indicators. Also, Pareto-rankings of both international and Russian observations was proposed as a non-compensatory measure of quality of life.[1]

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I. Quality of Life Concept

Economics, as well as all other sciences, puts the motto "Make life better" on its banner. What stands behind these words? What is "better"? What is "worse"? These questions lie in the normative rather than positive plane, and answering them requires certain bravity in dealing with eternal humanity problems.

The message of the word **"quality"** is compliance with previously formed judgements, or standards of comparison. If you have never met with a tynerator[1] you could never tell whether the one you were presented with is a qualitative one. So when one ponders upon quality, it means a comparison, putting objects side by side, finding the relevant attributes and ranking the differences between them.

The term "*Quality of Life*" relates to the description and evaluation of the nature or conditions of life of people in a certain country or region. Quality of life is formed by exogeneous, with respect to an individual or a social group, forces like production technology, infrastructure, relations with other groups or countries, institutions of the society, natural environment, and also by endogeneous factors including interaction within the society and values of a person or a society. The effect of these factors is not necessarily constant over time; for instance, environmental issues were paid relatively small attention a century ago while today ecology is undoubtedly one of the main people's concerns, and information technologies penetrating each shpere of human existense were not existent at that time at all.

The impetus to the development of the means of the quality of life assessment and evaluation was given in 1960s owing to the Social Indicators Movement. In 1960s, lots of public educational, social, ecological programs were initiated, and quantitative indicators to measure their success (or failure) were of great need. This movement, in turn, was brought about by a view in the society that life had in general become worse though the standards of living were considerably improving. This work resulted in a series of publications by Census Bureau (*Social*

[1] Do you know what is it? Neither do I.

Indicators, [1]) monitoring trends in society status, values, and people's lives quality.

Parallelly, the quality of life concept has been used in the medical literature for a long time to denote the state of the patient in the deviations from the psychic norm, during the after-operational reabilitation, or in oncologic diseases [2]. The two approaches differ in the object of investigation, the latter mainly focusing on the individual quality of life. We shall concentrate on social and economic aspects of the quality of life, i. e., we shall put society or economy to the center of our attention.

Two types of the quality of life measures, or indicators, are distinguished, namely, subjective and objective ones. Also, expert estimations which combine both subjective and objective approaches are also widely used.

The **subjective** indicators reflect subjective evaluations of people's lifes. They represent the microlevel of the quality of life data collected from the individual agents. The measurements of this kind are essentially personal and based on the individual reports of one's well-being and responses obtained in sociologic surveys and investigations [3]. People are asked what they feel about their life, its values, what they care about most, etc. The questionaries can sometimes contain up to a hundred of questions. Also, these data can be collected during a longitudional investigations of the population [4]. Subjective measures reflect both the real status of the quality of life, or the conditions of life in general, and the attitude of the people toward these conditions. The aggregation of these subjective measures by statistical techniques and methods can help identifying the values of society or different social groups.

The **objective** quality of life measures are built on the basis of "hard variables", i. e., the data from the municipal or governmental institutions and organizations which may include financial accounts, civil state records, medical statistics, pollution levels, and other pieces of information gathered by the institutions routinely. This approach aims at investigating the society as a whole by looking, in the most general sense, at the set of macroeconomic, social, demographic indicators which determine the conditions of life and the way people live.

As an objective measure, the quality of life may be defined as an interrelation of the four determinants of the vital functioning and activity of the population [5]. These are the quality of population, material welfare, the quality of the social system, and the quality of ecology, or environment. The hierarchical decomposition of every component can go down to the basic low-level characteristics assessed by the statistical data or by expert estimation. E. g., the *quality of population* is inferred from the population demographic structure, the reproductive process and marital behavior, the population physical, psychic, and moral health, educational level and proficiency. The *material welfare* is determined by the standards of living, income differentiation, housing, telecommunication, trade, education, culture, health system, mass media capacities. The *social system quality* relies upon the state and/or private provision in cases of the (permanent or temporary) disablement, citizens' rights for education, employment, recreation, private property and personal protection, political system stability, individual's inclusion to the social infrastructure, race and sex equality, social stability. The *ecology state* is influenced by the state of air and water basins, the level of chemical, radioactive, heavy metal pollution, etc. The list is far from being complete, and some items may be related to more than one category.[2]

So, each of the mentioned properties and measures, being expressed via a system of statistic indicators, should then be integrated into a measure of the overall quality of (the particular sphere of) life. It may also be required that the final aggregate measure satisfy the property of *uncompensatory preference*, i. e., shortcomings in one of the fields cannot be compensated by improvements in others. This requirement induces Pareto ranking in the phase space. Commonly used linear or log-linear weighted combinations do not satisfy this property.

The methodology of decomposition, or introduction of a "phase space" [6] is often used in related publications to assign the dimensions for factor scaling, with a more distant goal of multicriterial ranking [7]. A very similar approach is proposed in [8] where five integral indicators are proposed for estimating strategic decisions on regional level. Four of them next to coincide with earlier proposed factors and relate to demography, welfare, environment, and social comfort; the fifth indicator characterizes self-sufficiency of the region.

As another instance, Institute for Managerial Development (Lausanne) picks out eight main spheres of intercountry comparison [9]: domestic economy strength, internationalization, government, finance, infrastructure, management, research and development, and people. Each of the factors is then split down to specific criteria of the lowest level representing statistical data

[2] In this context it is worth reminding a buddhist parable [23] about blind men who tried to perceive what an *elephant* is. One touched the trunk and said, "An elephant is long and flexible". Another touched the tusk and said, "No, it is stern and sharp". Yet another stood under the belly and said, "It is somewhat tall and feels quite heavy" while the fourth stood behind holding the tail and concluded that an elephant was light and smelled ... hm ... not very pleasant. What did they miss? Wholeness: an elephant is all these things taken together.

or expert estimates; the book reports about 250 indicators in the eight fields cited above.

The provision of information for quality of life analysis requires collection both the data describing separate aspects of life and integral characteristics of the social and human existense in general. To correspond the problem of the quality of life improvement, the system of indicators is to be built on the hierarchy principle since monitoring of a large number of indicators (i. e., statistical data) seems to be an ungovernable task. The lowest level of this system consists of the sources, or basic statistical indicators, groupped according to separate fields of quality of life analysis. At upper level, this basic data are used to calculate the aggregate measures of the quality of life as a complex phenomenon. In the process of the aggregation, the unification of non-related and non-commensurable indicators is required which implies the use of expert and heuristic procedures. As a result, at the higher level of aggregation the estimates of the quality of life tend to be only relative measures, thus being referred to as integral estimates, or integral indicators, of the quality of life.

The expert estimation of the quality of life is carried out via comparison of the different countries, or different regions or cities of the same country, by the group of experts. The final data may be represented as an arbitrary numerical scale where the quality of life in the country is given some numerical value; the ordered scale where all countries are ranked in decreasing order, but without any numbers attached, or the matrix of pair comparisons where each two countries are compared. The case of numerical scale is the simplest one since it allows for regression models to be built. The two other types of data are substantially more difficult to be analysed [10, 12].

The literature on the subjective methods of the quality of life assessment will be overviewed in Section II.1. Section II.2 will concentrate on UN proposed measure of human development. Section III.1 will contain a simple regression analysis based on objective and expert-estimated variables. The necessary numerical data as well as mathematical methods description will be dealt with in Appendices.

II. Approaches To Quality of Life: An Overview

II.1 Subjective Evaluation: Data at Individual Microlevel

The subjective quality of life assessment has been studied in sociologic and, to some extent, economic literature for many years. The list of papers and books is quite exhaustive. The search system of the Library of Congress (`http://lcweb.loc.gov`) supplies about 1100 references for the request on the key words "`Quality of life`". The bulk of literature is dated by 1970s when the quality of life and other social issues were on the top of the public interest.

An examplary investigation to the quality of life is [13]. In this book the analysis of the quality of life perception by American citizens is carried out by using the US statistical data on both subjective evaluation of the quality of life in terms of happiness and satisfaction in most important areas of human life, and objective indicators of material well-being and individual resources. The semantic aspects of the response interpretation and corresponding significance of statistical inference is paid a lot of attention. Along with the major component identification (similar to the procedure of Section III.1), the time-trends in the general sense of well-being are also analysed. The peculiarities of the quality of life analysis for the social groups such as women and national minorities.

One of the most popular aggregate measures of the quality of life is the individual estimation of one's happiness. The concept of happiness has been paid attention by the leading thinkers since antique times. Modern approach, though, implies the accurate definition of the terms and construction of models for the applicable fields of study. According to this scheme, [14] distinguishes four main aspects of happiness inferred from the philosophic and scientific literature:

happiness as possession (epicurism), happiness as satisfaction of needs and requirements (utilitarism), happiness as self-realization (eudemonism), happiness as a positive result of comparison with other individuals and past experience (modern sociology). The results of empirical investigations conducted by the authors suggest that the degree of happiness is mainly explained, in the statistical sense, by the positive evaluation of life situations and beneficial comparisons with those of other individuals and past experience. Individual characteristic and resources at his disposal (gender, age, income) affect happinnes indirectly, via processes of evaluation and comparison.

Overwhelming are bibliographies [15] and [16] which index sociologic and philosophic literature related to subjective happiness. In the introduction to the latter source, concept of happiness is defined as follows: "Happiness is defined as the degree to which an individual judges the overall quality of her/his life as-a-whole favorably." This enormous book contains 2472 references on subjective life perception covering the period from the Socrate and Plato till the most modern studies. The author notes that the investigations on happiness were conducted in different branches of social sciences: in ideologic, philosophic, morale and modern scientific (sociologic) literature. The latter cluster of sources can hardly be analysed or even accompanied since this literature is not well documented and spread around in different fields of studies. Often happiness is a by-product of another research and, hence, not included into the title and keywords of a paper. Nevertheless, English, French and Danish literature is described quite extensively.

Another approach was used in the study "Cross-cultural quality of life research: an outline for a conceptual framework" by E. Hankiss, R. Manchin, and L. Fustos published in routine UNESCO publication [17]. A conceptual framework of economic flows and filters was outlined which determine how the actions of agents is reflected in the quality of life both objective and subjective. These filters are determined by institutions of the society and may include the values of society and individuals, functioning of the public distributional system, direction of time, life cycles, dominance of the society survival under conditions of exogenous influences (i. e., wars or natural catastrophes), the life goals of an individual.

I cannot go without mentioning the materials developed in our research group headed by Prof. S. A. Aivazian at Central Economics and Mathematics Institute of Russian Academy of Science (RAS CEMI). He has been interested in the problems of statistical analysis of income distribution, consumption behavior and consumer preference reconstruction, as well as other applications of mathematical statistics in social sciences, since he joined CEMI in late 1960s. Now our group is working on a project with Goskomstat (State Committee on Statistics, Russian Federation) which aims at developing a methodological framework for routine quality of life analysis in Russia in the course of Goskomstat surveys [5].

II.2 Human Development Index: An Aggregate Country-Level Measure

Human development index is a popular quantitative measure of the degree of a country's success in developing its human potential. Its introduction in early 90th was caused by the necessity to find the measure of human progress which is people rather than economic-centred. Economic growth does not necessarily imply human development, as well as a strong human potential is not always reflected in economic performance, but in general these factors act as mutually reinforcing entities, and this bilateral influence can additionally be magnified by proper government policies.

Since 1990, United Nations Development Programme (UNDP) has been publishing annual its *"Human Development Report"* [19] which explores in detail the relationship between human development and "hard variables" representing the economic, social, demographic state of the society. This program aims at revealing the relative performance of different countries via constructing a numerical measure of human development. Of course, the concept of human development is much deeper and richer than what can be captured in a composite index or even by a detailed set of statistical indicators. Yet it is useful to aggregate different aspects of a complex reality.

Though simple in construction, human development index (referred to as hdi further on) provides useful insights into the causes of the differences between countries' position in the world ranking, and even may be used as a policy target. It also reveals very sharply the structure and direction of the progress (or retrogression) in human potential in the course of economic growth of a country, and the problems accompanying this progress across approx. 170 countries for which necessary data are available.

The hdi index reflects achievements in basic human capabilities in three fundamental dimensions — a long and healthy life, an adequate education, and a decent standard of living (which are related, to some extent, to the factors contributing to the quality of life as introduced on page 5, namely, the material welfare, quality of population and social system quality). The variables representing these three dimensions are life expectancy, educational attainment and income. The hdi value for each country (region, etc.) indicates how far it has to go to attain certain defined goals: an average life span of 85 years, access to education for all and a standard of living on the world level. In fact, hdi weights deprivation in these three factors, i. e., the relative distanse from the desirable goal on the unit scale.

For any component of the hdi, individual indices are computed according to the general formula:

$$\text{Index} = \frac{\text{Actual value}_i - \text{Minimum value}_i}{\text{Maximum value}_i - \text{Minimum value}_i} \tag{II.1}$$

Fixed minimum and maximum values have been established for each of the indicators:

- Life expectancy at birth: 25 and 85 years;
- Adult literacy: 0% and 100%;
- Combined enrolment ratio: 0% and 100%;
- Real GDP per capita (PPP$): PPP$100 and PPP$40000.

Educational attainment index is built as a linear combination of adult literacy and combined primary, secondary and tertiary enrollment ratios with weights 2/3 and 1/3, respectively:

$$\text{Educational attainment} = \frac{2}{3}\,\text{Adult literacy} + \frac{1}{3}\,\text{Combined enrollment} \tag{II.2}$$

The construction of the income index is more complex and is based on the utility of income with varying elasticity:

$$W(y) = \begin{cases} y \quad \forall\, y:\, 0 < y < y^*, \\ y^* + 2[(y-y^*)^{1/2}] \quad \forall\, y:\, y^* \le y \le 2y^*, \\ y^* + 2(y^*)^{1/2} + 3[(y-2y^*)^{1/3}] \quad \forall\, y:\, 2y^* \le y \le 3y^*, \\ \text{etc.} \end{cases} \tag{II.3}$$

where y^* is a threshold level of income. For international comparison, y^* is taken to be the average world income. In [19] (1996) the value of PPP$5711 is cited as an average world income in 1996. Consequently, the utility of maximum (target) income, PPP$40000, or adjusted real GDP per capita, would be

$$W(40000) = y^* + 2(y^*)^{1/2} + 3(y^*)^{1/3} + 4(y^*)^{1/4} + 5(y^*)^{1/5} + \\ 6(y^*)^{1/6} + 7(y^*)^{1/7} + 8[(40000 - 7y^*)^{1/8}] = \text{PPP\$6040}.$$

The hdi is an average of the life expectancy, educational attainment and the adjusted real GDP per capita (PPP$) indices, i. e., the sum of the indices divided by three:

$$\begin{aligned} HDI = {} & (\text{Life expectancy index} + \text{Education attainment index} + \\ & \text{Adjusted income index})/3 \end{aligned} \tag{II.4}$$

Thus this is a composite comparative index reflecting relative performance of a country or a region as compared to some fixed standards.

The methodology of the hdi calculation is very young and still in the process of formation. For instanse, [19] (1995) notes: "Since *Human Development Report 1994*, two changes have been made in the construction of the HDI relating to variables and minimum and maximum values. First, the variable of mean years of schooling has been replaced by the combined primary, secondary and tertiary enrolment ratios, mainly because the formula for calculating mean years of schooling is complex and has enormous data requirements ... Second, the minimum value of income has been revised from PPP$200 to PPP$100". Threshold level of income y^* is also changing with time. So it makes sense to state explicitly which data, maximum and minimum values, threshold levels were used in computing hdi.

Gender-Related Development Index

The methodology of hdi calculation allows for adjustment for inequality in positions of different population groups, e. g., men and women. The general assumptions underlying the construction of gdi (gender-related development index) are those of equality in potentials of the two group except for the longevity which is known to be higher for women, and of social preference for equality represented by a specific weighting formula. Another assumption should also be made related to the sources of income when considering gender income discrimination that the ratio of non-labor income to labor income is the same across population.

With gender-adjusted maximum and minimum life expectancy values (87.5 and 27.5 years for women; 82.5 and 22.5 years for men), the same formulas (II.1)–(II.4) are used to calculate partial men and women related partial indices.

Different approaches could be taken to account for the gender difference obtained on the previous step. The simplest one is to multiply the hdi by the ratio of the female hdi to the male one. This used to be an early approach found in [19] (1993). Later, it was substituted to a more complicated one.

Since 1995, UNDP have been exploring the extension of the approach of [20] with risk, or inequality, aversion. Let X be an indicator of achievement, and let X_f and X_m refer to the corresponding female and male achievements. If n_f and n_m are the numbers of females and males in the population, the overall or mean achievement $\bar{X}$ is given by

$$\bar{X} = (n_f X_f + n_m X_m)/(n_f + n_m) \tag{II.5}$$

This mean achievement is compared to an "equally distributed equivalent achievement" X_{ede} defined as the level of achievement which is socially equivalent to the actually observed vector (X_f, X_m). Social valuation is defined by the concave function with constant elasticity:

$$V(X) = \begin{cases} \dfrac{X^{1-\epsilon} - 1}{1-\epsilon}, & \epsilon \geq 0, \epsilon \neq 1 \\ \ln X, \epsilon = 1 \end{cases} \tag{II.6}$$

X_{ede} is thus defined through

$$(n_f + n_m)\frac{X_{ede}^{1-\epsilon}}{1-\epsilon} = n_f \frac{X_f^{1-\epsilon}}{1-\epsilon} + n_m \frac{X_m^{1-\epsilon}}{1-\epsilon} \tag{II.7}$$

or, after simplification,

$$X_{ede} = (p_f X_f^{1-\epsilon} + p_m X_m^{1-\epsilon})^{\frac{1}{1-\epsilon}}, \tag{II.8}$$

where $p_m = n_m/(n_f + n_m)$, $p_f = n_f/(n_f + n_m)$ — proportions of male and female in total population. In other words, X_{ede} is a $(1-\epsilon)$ average of X_f and X_m: *arithmetic* for $\epsilon = 0$, *geometric* for $\epsilon = 1$, *harmonic* for $\epsilon = 2$ which is the value used in UNDP calculations. When $\epsilon \to \infty$, $X_{ede} \to \min(X_f, X_m)$. The index of relative equality E that underlies X_{ede} can be defined as

$$E = X_{ede}/\bar{X} \tag{II.9}$$

while the corresponding measure of relative inequality I is simply the Atkinson index:

$$I = 1 - X_{ede}/\bar{X} = 1 - E \tag{II.10}$$

The resulting X_{ede} for the life expectancy, educational attainment and income are then used to calculate the gender-related development index.

The main obstacle for this methodolgy lies in absense of precise data for gender differences in earnings and rewarded employment. Gender-specific attributions of income per head cannot be readily linked to the aggregate GDP per capita used in standard hdi calculations. Moreover, inequalities within the household are difficult to characterize and assess since the household is usually viewed as a whole in the aspect of money *use* while men and women can differ significantly by the *earnings* aspect of income, and this difference can be estimated based on statistical data (world average ratio of female to male wages is about 75%). Besides, the work inside the household remains unpaid and not accounted for in GDP. So the human development methodolgy faces a trade-off: it is easier to digitize the employment and compensation figures while the entire approach of human development concept has been based on what people get out of the means they can use.

Another subtle point is the choice of the inequality aversion degree ϵ. For value of zero, there is no decline in marginal values, and inequality is in fact neutral to the hdi measure. When ϵ is infinity, however, the achievement of the better-off is ignored. The key point is the effect of ϵ on X_{ede}, e. g., in the effect of unit increase in separate achievement on X_{ede}. It may be shown that

$$\frac{\partial X_{ede}/\partial X_f}{\partial X_{ede}/\partial X_m} = \left(\frac{X_m}{X_f}\right)^{\epsilon} \tag{II.11}$$

assuming that $p_f = p_m = 1/2$ and constant elasticity of social valuation.

III. Quality of Life Measurements On The Real World Data

The final result of my work is supposed to be the linkage between the objective, subjective and/or expert-estimated measures of the quality of life. This would be valuable for estimating and forecasting the quality of life on the basis of the hard variables which can be gathered from the government organizations and institutions.

There may be different ways to approximate the statistically dependent variable (here, quality of life) by the set of independent variables, even among parametric models, and even among linear models in the form

$$\mathsf{qol} = \mathbf{W}\,\mathbf{X} + \epsilon \tag{III.1}$$

By stating different optimization problems, one can obtain different estimates of the weights $\mathbf{W}$:

$$[\mathsf{qol} - \mathbf{W}\,\mathbf{X}]^2 \to \min \tag{III.2}$$

$$\mathrm{Var}(\mathbf{W}\,\mathbf{X}) \to \max \tag{III.3}$$

$$\mathrm{corr}(\mathsf{qol}, \mathbf{W}\,\mathbf{X}) \to \max \tag{III.4}$$

Here, equation (III.2) corresponds to the standard ordinary least squares regression estimation method, (III.3), to the *principal components* method (see Appendix C), and the equation (III.4) can be described as maximum similarity approach, where correlation operator $\mathrm{corr}(\cdot,\cdot)$ may be Pearson correlation, as well as rank correlation measures (see Appendix D). In the latter case, the results are likely to be more robust and relatively "soft" with respect to deviations in the data. It will be demonstrated that all these approaches lead to surprisingly similar and good results.

To identify the quantitative relations of quality of life with main economic and social indicators, I have been working with the data from Institute for Managerial Development (IMD) [9], an international organization of World Economic Forum, Lozanne. A detailed introduction to this source and the notation of the variables used further are given in Appendix A. These data are valuable since they contain quality of life expert estimates and thus may be used as a training sample to develop a methodology for the analysis which can be later applied to other data, e. g., to analyze the quality of life across regions of Russia.

III.1 A Simple Regression Model

In this section, a standard least-squares regression approach was implemented to identify a linear relation between the expert estimates of quality of life for different countries and the factors that determine this quality.

The pool of regressors was formed according to the four group of factors specified in the Section I. For such a small sample, it would have been useless to work with more than three or four regressors from each of the groups due to the lack of degrees of freedom. The final regression was expected to contain from one to three factors from each group.

To identify the most important factors, the step-wise regression approach was used (see Appendix B). It could also be interesting to use the only "hard variables" (i. e., the real economic and socio-demographic data) and not take into account expert estimates of different type.

Year 1995 data.

The process of the regressors selection went as follows:

No.	Regressors					R^2_{adj}	SBIC
	const	PPP	POLIT	EDUC	LCO2		
0	5.87 (2.08)						
1	2.88 (0.37)	0.246 (0.027)				0.645	0.554
2	1.01 (0.56)	0.214 (0.024)	0.498 (0.123)			0.736	0.318
3	1.15 (0.57)	0.177 (0.041)	0.496 (0.122)	0.480 (0.043)		0.737	0.372
4	3.18 (0.42)	0.289 (0.039)			0.48 (0.32)	0.654	0.585

The last line corresponds to the regression in "hard variables".

The data are multicollinear which is evidenced by the drift of the ppp coefficient until it becomes insignificant. Also, this multicollineraity is easily seen from the correlation matrix:

	PPP	POLIT	EDUC	LCO2
QOL	0.808	0.552	0.715	-0.501
PPP	1.000	0.322	0.825	-0.732
POLIT		1.000	0.271	-0.178
EDUC			1.000	-0.585

It is worth noting that the regression model gives a good description of the dependent variables not only in levels but in rankings, too. The correlation coefficients for the equation 3 of the Table above are: Pearson correlation, 0.8685; Kendall rank correlation, 0.6633; and Spearman rank correlation, 0.8640. All are significant at 0.1% level.

Year 1996 data.

Here, the regressor selection was somewhat different:

No.	Regressors						R^2_{adj}	SBIC
	const	JUSTC	LCO2	HDI	POLIT	PPP		
0	6.43 (2.20)							
1	2.27 (0.54)	0.748 (0.089)					0.647	0.672
2	8.64 (1.57)	0.582 (0.084)	0.818 (0.194)				0.758	0.365
3	1.15 (0.57)	0.177 (0.041)	0.496 (0.122)	0.480 (0.043)			0.737	0.372
4	2.18 (2.34)	0.491 (0.078)	0.701 (0.174)	7.10 (2.08)			0.813	0.171
5	1.25 (2.10)	0.333 (0.085)	0.786 (0.157)	8.07 (1.87)	0.340 (0.106)		0.852	0.009
6	1.84 (2.07)	0.245 (0.096)	0.667 (0.166)	5.95 (2.17)	0.400 (0.108)	0.055 (0.031)	0.861	0.033

The results are somewhat more advanced than before: more regressors are involved in explanation of the quality of life as well as better statistical accuracy of the model is obtained. Nevertheless, the results are rather different for the two years what can lead to a thought that the techniques for data collection within IMD were different, or different experts were enrolled in estimating quality of life and other non-hard data. To minimize subjective interference, "hard variables" regressions may be considered:

No.	Regressors					R^2_{adj}	SBIC
	const	PPP	LCO2	GREEN	PHYS		
0	6.43 (2.20)						
1	3.87 (0.42)	0.203 (0.028)				0.572	0.868
2	9.37 (2.07)	0.148 (0.332)	0.723 (0.267)			0.634	0.777
3	11.92 (1.92)	0.052 (0.039)	1.227 (0.269)	0.782 (0.216)		0.726	0.552
4	14.30 (2.16)	0.013 (0.042)	1.49 (0.29)	0.833 (0.207)	−0.354 (0.169)	0.750	0.525

The data are again multicollinear:

	JUSTC	LCO2	HDI	POLIT	PPP	GREEN	PHYS
QOL	0.811	0.679	0.636	0.517	0.763	0.500	-0.221
JUSTC	1.000	0.472	0.454	0.548	0.659	0.438	-0.207
LCO2		1.000	0.369	0.110	0.615	-0.034	0.155
HDI			1.000	0.103	0.689	0.544	-0.542
POLIT				1.000	0.116	0.358	0.044
PPP					1.000	0.513	-0.289
GREEN						1.000	-0.260

It is worth noting that political factor is only weakly correlated to other measures.

The predictive power of the model in terms of rankings is again very high. Pearson correlation between qol and the regression 6 is 0.9377, and rank correlation Kendall and Spearman coefficients are 0.7095 and 0.8778, respectively, with significance level below 0.1%.

The exclusion of the outliers (Russia, China, Czech Republic, India, with too high CO_2 emissions, Brazil and South Africa, with too high income inequality, China, Philippines, Indonesia with too low human development index) did not change the procedure and results qualitatively though allowed for more regressors to be included in the model.

The results may be explained as follows: the difference in the quality of life between main groups of countries (OECD members, developing and CEE countries) is in essense explained by the per capita income in purchasing power parity terms. The factors of quality of population are also important. Within the group of countries, the explanation mechanisms are not so clear, one of the main class of factors being environmental ones. Political instability in the country, as well as lack of education, can deteriorate the quality of life quite significantly.

The social system factors were not adequately presented in this survey since the data from [9] contained very little and non-systematic information about this sphere. Nevertheless, it was decided not to deviate too far from this source because of its comprehensiveness and general reliability.

III.2 Principal Components Analysis

Another approach to reduction of dimensionality is the method of principal components [10] described in detail in Appendix C.

The following data from the initial pool of variables were taken into account: PPP, EDUC, HDI, POLIT, EXPCT, logs of SECUR, CO2, INEQ, URBAN. In year 1995, data for telecommunications and recycling rate were also used. These data were standardized according to the generaliztion of rule C.3 so that value of 1 corresponds to the "best" possible case and 0, to the worst[1]. The "best" value for the inequality of income is defined as the average of the five countries with the highest quality of life in the sample, namely, Norway, Canada, Switzerland, Australia, and New Zealand.

The principal components identification was carried out in the SPSS for Windows v. 6.1 package and lead to the following results.

Year 1995 data

In the table below the three first principal components are given which correspond to the eigenvalues greater than one.

Factor	Eigenvalue	% of variation	Cumulative % of variation
1	6.40202	53.4	53.4
2	2.14912	17.9	71.3
3	1.04632	8.7	80.0

The coefficients of the variables in the factors were:

	Factor 1	Factor 2	Factor 3
SEDUC	0.85229	-0.05989	0.09464
SEXPECT	0.82878	-0.38658	0.23688
SHDI	0.74874	-0.55255	0.08146
SLCO2	0.73842	-0.52681	-0.15805

[1]Standardization is denoted by the prefix S below.

	Factor 1	Factor 2	Factor 3
SLGREEN	0.64726	-0.44055	-0.35944
SLINEQ	0.36970	0.23153	0.75247
SLSEC	0.61833	0.64024	0.14499
SLURB	0.75227	0.44326	-0.25943
SPOLIT	0.53135	0.55685	-0.39319
SPPP	0.93932	-0.16194	0.09774
SRECYCL	0.83114	0.25837	-0.03829
STELECM	0.72356	0.39510	-0.01708

Besides, the analysis for the "hard data" was carried out:

Factor	Eigenvalue	% of variation	Cumulative % of variation
1	5.10568	63.8	63.8
2	1.14019	14.3	78.1

	Factor 1	Factor 2
SEDUC	0.84520	0.21401
SEXPECT	0.90958	-0.02283
SHDI	0.87486	-0.23712
SLCO2	0.84561	-0.36694
SLGREEN	0.73529	-0.39413
SLINEQ	0.32577	0.76403
SPPP	0.95029	0.12278
SRECYCL	0.73056	0.38580

In the end, the first principal components in both cases were compared to the QOL and correlations were calculated:

	Enlarged set	Hard data
Pearson	0.8482	0.7781
Kendall	0.6365	0.5727
Spearman	0.8457	0.7871

The significance level of these correlation coefficients was at least 0.001 (SPSS showed it as

.000).

The results show a good correlation between rankings as derived from the expert estimate of the quality of life, on one hand, and the principal components of the available data. Most variation is explained by the first principal component. The result is almost a striking one. It implies that quality of life is generally determined by the exogeneous conditions of life given by "hard variables".

Year 1996 data

For this year, the results were as follows:

Factor	Eigenvalue	% of variation	Cumulative % of variation
1	5.24760	52.5	52.5
2	1.41421	14.1	66.6
3	1.26290	12.6	79.2

	Factor 1	Factor 2	Factor 3
SEDUC	0.87077	-0.05640	0.17121
SEXPECT	0.90837	-0.23492	0.12007
SHDI	0.77938	-0.49420	-0.04092
SLCO2	0.62766	0.26852	0.45706
SLGREEN	0.62341	-0.48957	-0.45708
SLINEQ	0.44284	0.13045	0.36249
SLSEC	0.67878	0.66243	0.13978
SLURB	0.74948	0.27824	-0.38617
SPOLIT	0.36720	0.50197	-0.70365
SPPP	0.95958	-0.12083	0.06692

It is easily seen that politic environment is least correlated with the first principal component which supports the statement made earlier (see p. 19) of relatively weak correlation of the political factor with other variables.

Again, "hard data" based components were determined:

Factor	Eigenvalue	% of variation	Cumulative % of variation
1	4.33389	61.9	61.9
2	1.01567	14.5	76.4

It is interesting to note that despite reducing the number of variables taken into account the predictive power of the first component has grown as measured by the variation explained (% of variation).

	Factor 1	Factor 2
SEDUC	0.87086	0.14221
SEXPECT	0.93935	-0.01384
SHDI	0.86057	-0.31928
SLCO2	0.61542	0.47391
SLGREEN	0.67223	-0.59365
SLINEQ	0.43945	0.56184
SPPP	0.96374	0.02534

Finally, correlations were again found to be statistically significant, if not strong:

	Enlarged set	Hard data
Pearson	0.8942	0.8196
Kendall	0.7009	0.6105
Spearman	0.8840	0.8199

III.3 Application to Regions of Russia

On the basis of the previous analysis, the empirical study for Russian regional data was carried out. The data are described in Appendix A, p. 32.

Unfortunately, the data did not contain the education enrollment figures which prevented us from computing HDI index for regions of Russia.

To analyse the principal components of this data set, the data were standarsized according to the rule (C.3). The principal components analysis went as follows:

Factor	Eigenvalue	% of variation	Cumulative % of variation
1	4.57974	35.2	35.2
2	2.46205	18.9	54.2
3	1.47313	11.3	65.5

Evidently, the first principal component for Russian data set desribes less variance than that for international data does. This could be attributed to the great dispersion of main important characteristics like geographic position, population density, resource base, etc. which can vary in an order or more of magnitude from say Tambov region to Yakutia.

Even more disappointing were negative correlation between the first component and the factors in question. It signals that the first component seems to be non-appropriate measure of quality of life for Russia. As a poet said, "Russia cannot be measured with a common ruler".

	Factor 1	Factor 2	Factor 3
SCRIME	0.56903	0.29041	-0.56373
SDOC	-0.42572	0.48440	0.28692
SDWELL	-0.27052	0.76271	0.06008
SEMISSC	0.73695	0.11342	0.57715
SEMISSD	0.69397	0.26316	0.36735
SEXPECT	0.75117	0.32498	-0.40922
SGDP	-0.70324	0.14873	-0.38326
SILL	0.51907	-0.11269	0.17854
SMURDER	0.50828	0.64832	-0.30042
SNURSE	-0.56748	0.51362	0.27727
SROADS	0.61045	0.48644	-0.01269
STELE	-0.62772	0.14088	-0.23013
SUNEMPL	-0.54913	0.63101	0.18675

The results could also be in part understood by significantly smaller correlations between the variables. For instanse, in the correlation matrix of the size 14×14, only 7 correlation coefficients were greater in absolute values than 0.5 (or 7.7%), while for the international data set, there were 44 of such correlations in 22×22 matrix (19.1%). This less developed multicollinearity increase estimates efficiency but at the same time might point to less reliable data: they are supposed to be more tightly related, and smaller variance could indicate somewhat more stochastic data generation.

With a narrower set of variables corresponding to those used in international analysis, the results were generally the same:

Factor	Eigenvalue	% of variation	Cumulative % of variation
1	2.85545	40.8	40.8
2	1.72314	24.6	65.4

	Factor 1	Factor 2
SDOC	0.55743	0.53512
SEMISSD	-0.61177	0.42388
SEXPECT	-0.72902	0.50095
SGDP	0.62132	0.07149
SMURDER	-0.49341	0.75630
SNURSE	0.73608	0.43406
SUNEMPL	0.68386	0.49056

Finally, the resulted rankings of regions of Russia are given in Appendix F. They can hardly be expected to give precise estimations of the quality of life across Russia due to the statistical (and intuitive) reasons given above though to some extent they capture some trends in economic and social performance.

III.4 Pareto classification

The measures of quality of life proposed above are linear by construction and thus do not satisfy the non-compensatory principle proposed in Introduction. It requires that lower levels of any factors cannot be compensated by higher levels of others.

As the first approximation to the non-comepsatory quality of life indicators, Pareto classification (see Appendix E) is proposed. The methodology allows to compare the observations (here, countries or regions) by all of the factors taken together, and not by any aggregate measure. The main application of such a classification viewed so far is to determine the groups of countries or regions with comparatively higher/lower levels of achievements. This supports an intuititve idea of existense of clusters with more or less homogeneous quality of life that differ significantly from that of objects from other clusters. For intracountry region comparison, the results of classification may serve as a basis for region policy design and conduction.

Pareto classification has been conducted for both international and Russian data sets (all available data and most relevant factors). For international IMD data, analysis was carried out for years 1995 and 1996, taking into account the larger set of factors, (sppp, slsec, seduc, slco2, slurb, sexpect, slineq, shdi, spolit, srecycl, slgreen, stelecm, see data description) as well as factors selected by the stepwise regressions (see Section III.1).

In year 1995, Pareto classification on the complete set of factors revealed two groups:

1. Argentina, Australia, Austria, Belgium & Luxemburg, Brazil, Canada, Chile, China, Colombia, Denmark, Egypt, Finland, France, Germany, Greece, Hong Kong, Iceland, India, Indonesia, Israel, Japan, Jordan, Malaysia, Mexico, Netherlands, New Zealand, Norway, Peru, Portugal, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, USA

2. Czech Republic, Hungary, Ireland, Italy, Korea, Philippines, Russia, South Africa, Taiwan, Venezuela

For the narrower set of factors selected by the stepwise regression (ppp, polit, educ, lco2), there were 5 distinct groups found:

1. Argentina, Austria, Brazil, Chile, Colombia, Denmark, Egypt, Finland, Germany, Hong Kong, India, Indonesia, Jordan, Malaysia, New Zealand, Norway, Peru, Portugal, Singapore, Spain, Sweden, Switzerland, Thailand, USA

2. Australia, Canada, China, Czech Republic, France, Hungary, Iceland, Ireland, Japan, Korea, Mexico, Netherlands, Taiwan, Turkey

3. Belgium & Luxemburg, Israel, Italy, United Kingdom, Venezuela

4. Greece, Philippines, South Africa

5. Russia

Larger number of groups is absolutely in compliance with the suggestion of Appendix E on the relation between the dimensionality of the phase space, the number of observations, and the groups identified.

Interestingly, Australia with the highest expert estimate of the quality of life was classified to the 2nd group. The detailed analysis reveals that it has relatively weak positions in education expenditures as a fraction of GDP (rated 0.38 on [0,1] comparative scale). That was enough to scale it down in the classification, and that is the essense of non-compensatory principle.

Evidently, Russia had serious problems in most of the factors analysed. In fact, all ratings were below 0.15 on unit scale. That did not allow to compete even with countries with lower income per capita in PPP$ like Philippines and South Africa.

For the year 1996 data, three classifications were carried out: on the complete set of 10 factors (sppp, slsec, seduc, slco2, slurb, sexpect, slineq, shdi, spolit, slgreen), and the two sets of factors selected by the stepwise regression: i) JUSTC, LCO2, HDI, POLIT, PPP; ii) "hard variables" PPP, LCO2, GREEN, PHYS.

The classification according to the 12 factors again revealed two groups that did not differ too much from the 1995 results:

1. Argentina, Australia, Belgium, Canada, Chile, China, Denmark, Finland, France, Germany, Hungary, Iceland, India, Indonesia, Israel, Japan, Korea, Malaysia, Netherlands, New Zealand, Norway, Philippines, Portugal, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United States

2. Brazil, Colombia, Czech Republic, Ireland, Italy, Mexico, Poland, Russia, South Africa, Venezuela

The classification according to the most relevant factors was as follows:

1. Belgium, Canada, Denmark, Finland, France, Iceland, Indonesia, Israel, Japan, New Zealand, Norway, Singapore, Sweden, Switzerland, United States

2. Australia, Chile, Czech Republic, Germany, Ireland, Italy, Malaysia, Netherlands, Spain

3. Argentina, Brazil, Portugal, South Africa, United Kingdom

4. China, Colombia, Hungary, India, Korea, Philippines, Thailand, Turkey

5. Mexico, Poland, Venezuela
6. Russia

The last classification according to the most relevant “hard variables” also was detailed enough:

1. Argentina, Belgium, Chile, China, Hungary, India, Indonesia, Israel, Italy, Japan, Korea, Philippines, Portugal, Spain, Sweden, Switzerland, USA
2. Canada, Czech Republic, Denmark, France, Germany, Iceland, Mexico, Norway, Russia, Turkey, Venezuela
3. Australia, Colombia, Finland, Netherlands, New Zealand, Poland, Singapore, South Africa, Thailand
4. Brazil, United Kingdom
5. Ireland
6. Malaysia

Relatively high place of Russia (as well as countries like Turkey or Venezuela usually rated rather low in quality of life estimates) can be explained by subjective factors like political instability or ideologic barriers still in place. Besides, an expert opinion on these countries may also be biased by the common fears of “bears in the streets”.

Pareto classification of regions of Russia was also carried out for two sets of factors: the complete set of data chosen for quality of life analysis, and one-factor-per-factor-group set (GRP, life expectacny, population per doctor, dwelling per capita, emissions). The resulting table is given in the Appendix F and by far more successful than the ranking according to the first principal component.

IV. Conclusion

The Quality of Life concept has not yet attracted much attention from the policymakers as a policy goal to be pursued. Nevertheless, the analysis of the quality of life as a measure of comparative performance of the society and its evaluation by experts or by the population itself is of interest. For this analysis, the relation between the quality of life estimations and real economic variables measured by the official institutions or predicted by macroeconomic models is to be established.

This Master Thesis attempted to outline the ways of identifying these relations. The hierarchical aggregate quality of life estimate decomposition to the four groups of factors was proposed. A simple regression analysis of the quality of life in different countries in 1995 on the factors from these groups was conducted. It was found that the difference in the quality of life between different countries is well explained by such factors as the GDP per capita in purchasing power parity terms, the level of security in the country, political system adequacy, and the state of ecology. Each of the groups of hierarchically decomposed factors were represented by one or two regressors.

Also, principal components of the main indicators of economic and social well-being of a society were found to be significantly correlated with the expert estimates of the quality of life and thus may serve as practically implementable method of the quality of life estimation. Such implementation was done on the Russian regions data.

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I claim copyright on all errors and omissions.

A. Data

The main source of the data for this research was [9] by International Institute for Managerial Development, Lausanne, which is an institution by World Economic Forum. This book is published annually and contains the data on more than 40 countries (OECD and other more or less developed countries, Russia included, sans Gulf region). There is a detailed report on each of the countries and separate tables for each of the indicators. All factors are collected into 8 groups: domestic economy, internationalization of economy, government, financial sector, infrastructure, management, science, people. The data and the very indicators tend to differ from year to year, so that no common reference to the Table's numbers is possible, but nevertheless the most important data fortunately did enter the book in all year analysed under the same name[1]. There are both real data ("hard variables") and expert estimation on the (0,10) scale where 0 corresponds to the lowest imaginable level, and 10, to the best one.

There is an expert estimation of the quality of life (Table 8.33 in [9], 1996) which was extensively used in this study for empirical tests. Some other data were considered relevant to the quality of life analysis and used in multivariate analysis and modelling as described below:

QOL Quality of life, an expert estimation on (0,10) scale; 10 for adequate quality of life, 0 for inadequate.

Welfare factors group:

PPP Gross domestic product per capita (US$ at current prices and purchasing power parity)

GDP Gross domestic product at constant prices (US $ billions at current prices)

INEQ Inequality of income distribution, the ratio of the wealthiest 20% of population income to that of the 20% poorest

[1] It might be suspected that the methodology of data collection did change from year to year as some descriptive statistics evidenced.

INFL Inflation, % per year

URBAN Urbaniztion influence, expert estimate: 0, cities drain national resources; 10, cities support national development

Quality of population group:

EXPCT Life expectancy at birth, years

UNEMPL Unemployment, % of labor force

ILLIT Adult (over 15) illiteracy rate, % of population

HDI Human development index, Section II.2

JUSTC Expert estimation of justice in country; 10 for adequate justice system, 0 for inadequate

CRIME Serious crime: number of murders, violent crimes or armed robberies reported per 100 th. inhabitants

SECUR Security in the country, expert estimation: 0, no confidence among people that their person and property is protected; 10, full confidence

COMPT Attitude towards competitivenes, expert estimation: 0, values of the society do not support competitiveness (such as pursuing individual interest at the expense of company interest); 10, support competitiveness (such as hard work, tenacity or loyalty)

POLIT Political system, expert estimation: 0, needs serious restructuring; 10, is well up-to-date for todays economic challenges

TAXES Personal taxes influence, expert estimate: 0, discourage individual work initiative; 10, encourage individual work initiative

Quality of environment:

CO2 CO_2 emissions from industrial processes in metric tons per capita

GREEN Greenhouse index, carbon heating equivalents in metric tons per capita

Social system adequacy:

CONTR Contribution to the social security funds, the sum of employee's and employer's contribution, % GDP

EDUC Education expenditures, % GDP

PHYS Population per physician

NURSE Population per nurse

Some of the indicators may be considered as applicable to more than one group. E. g., URBAN may be placed into environmental group, etc.

The set of the data used for the analysis is available on the Internet on the NES site.
`http://www.nes.cemi.rssi.ru/~skolenik/qol/mthesis/comp9596.xls`

Data for Russia were taken from the official issue of GOSKOMSTAT (State Committee for Statistics of Russian Federation) [22]. This two-volumed book contains a comprehensive set of official statistical data for 81 regions of Russia, including data for national autonomies, if necessary and available. These data were collected from statistical agencies which in turn achieve them from enterprises and instiutions as well as by censuses and surveys; from Russian ministries and governmental organizations, and from the third party agencies conducting surveys and analyses of economic and social issues. Most data are dated by year 1995; there is also (preliminary) data for year 1996.

From about 250 statistical tables, the following data were used for the analysis:

GDP GDP per capita, th. rb.

EXPECT Life expectancy at birth, years

UNEMPL Unemployment rate, %

ILL Illness rate, newly diagnosed per 1000 population

DOC Population per physician

NURSE Population per nurse

DWELL Dwelling per capita, sq. m.

CRIME Registered crimes per 100000 of population

MURDER Murders per 100000 of population

EMISSC CO_2 emissions, tons per capita

EMISSD CO_2 emissions, tons per $ 1000 of GDP

TELE Telecommunication services, rbs. per capita

ROADS Density of motorways, km/sq. km

These data are also available on the Internet via
`http://www.nes.cemi.rssi.ru/~skolenik/qol/mthesis/rrus.xls`

B. Stepwise regression

The method of stepwise regression is used to identify a regression model, or, equivalently, to select the most appropriate variables to explain the dependent one. At the first step, the regressor with the highest correlation with the dependent variable was selected to be included in the model. On the following steps, the variable is chosen which explains the greatest part of variation of the residual from the previous step regression as signalled by the partial correlation with fixed other variables (already selected at the earlier steps), or correlation between the residual and the considered variables from a pool of regressors. Let us denote y, the dependent variable; $x_1, \dots, x_k$, the regressors chosen on earlier k steps; $\tilde{x}_{k+1}^{(k)}, \dots, \tilde{x}_n^{(k)}$, the pool of regressors. Then the process goes as follows:

1. Calculate the residuals ε of the explaining regression:

$$y = \hat{a}_1^{(k)} x_1 + \ldots + \hat{a}_k^{(k)} x_k + \varepsilon^{(k)} \tag{B.1}$$

2. Find the correlation coefficients of $\varepsilon^{(k)}$ and $x_i^{(k)},\ i = k+1, \dots, n$.

3. Choose the regressors with the greatest correlation:

$$x_{k+1} = \operatorname{Arg}\max_j \operatorname{corr}(\varepsilon^{(k)}, x_j^{(k)}),\ j = k+1, \dots, n \tag{B.2}$$

The process terminates when the regressor chosen is not significant. There may also be other measures for model identification, e. g., Shwartz Bayesian information criteria (SBIC) which shows the trade-off between the model accuracy and the number of regressors, or R^2_{adj}.

C. Principal components

The principal components method[1] is used to reduce the dimensionality of the set of statistically processed data, e. g., for visual purposes or in order to obtain a low-dimension model. In this reduction, from the initial set of data $x_i^{(1)}, \ldots, x_i^{(p)}$, $i = 1, \ldots, n$, where p is the number of indicators, and n, number of observations, a new set of variables $z^{(1)}, \ldots, z^{(p')}$, $p' < p$ (or even $p' \ll p$) is obtained as an $\operatorname{Arg\,max} I_{p'}(Z(X))$ for some information measure $I_{p'}(\cdot)$.

If one is looking for the set $z^{(1)}, \ldots, z^{(p')}$ allowing only for linear transformations of the x's and maximizing the explained variance of the x's

$$I_{p'}(Z(X)) = \frac{\operatorname{Var} z^{(1)} + \ldots + \operatorname{Var} z^{(p')}}{\operatorname{Var} x^{(1)} + \ldots + \operatorname{Var} x^{(p)}} \tag{C.1}$$

then the resulting z's are called *principal components*, i. e., the factors that change most while switching between observations.

Principal components are used to restore the values of the variables from the least possible information set. For instance, the principal components of one's body may be inferred from the usually measured height, waist, neck size, etc., which are used to select a dress or a suit. Nevertheless often this information is redundant since these characteristic are to some extent interrelated: one seldom has short legs if he is 6 feet 5 inches tall. The idea of reduction of such redundancy is exploited in the principal components method. Sometimes, one figure is enough to identify the whole variety of the data if the scatter diagram has the only vividly evident axis.

Principal components depend only on the variance-covariance matrix $\operatorname{Var} x$ so they are invariant with respect to linear shift of variables $x^{(j)} \to x^{(j)} - c(j)\ \forall c^{(j)}$. Further we shall assume that the data are *centered*: $\mathbf{E}\, x^{(j)} = 0$.

It can be shown that principal components may be found as eigenvectors of the variance-covariance matrix. The first principal components (i. e., the linear combination which explains

[1] The description follows [10]

the major part of variance in the set of x's) corresponds to the largest eigenvalue of the variance-covariance matrix. Other components correspond to the consequently decreasing eigenvalues.

The use of the principal components method is quite natural when all components of the X vector are of the same nature and are expressed in the same units. If they are not, the result would depend heavily on the scale of the variables. Hence it makes sense to standardize them. Two normalization are generally used:

$$x^{*(j)} = \frac{x^{(j)} - \mathbf{E}\, x^{(j)}}{\sqrt{\operatorname{Var} x^{(j)}}} \tag{C.2}$$

$$x^{**(j)} = \frac{x^{(j)} - \min x^{(j)}}{\max x^{(j)} - \min x^{(j)}} \tag{C.3}$$

The transformation (C.2) leads to the set of variables with zero mean and unit variance, while the transformation (C.3) reduces all variables to the [0,1] range.

D. Rank correlation

The concept of rank correlation[1] appears in the analysis of statistical relationship between ordinal variables, e. g., ranks of other variables or rank of objects derived by expert estimation, which show the degree of maturity of the analysed characteristic. It is useful when the quantitative measure of the characteristic is not available or has only a relative sense as a tool for further ranking. It is also useful as a robust measure of interdependence of the two variables (though the concept of rank correlation can be generalized to the multivariate case).

Hence, the object of the analysis are labels indicating the rank $x_i^{(k)}$ of the object i in the total range of objects ($i = 1, \dots, n$) by the k-th characteristic, ($k = 0, \dots, p$).

The specific feature of an ordinal random variable is varying domain of possible values: the length of the range is determined by the very sample size and necessarily grows with it (while for cardinal random variables, the distribution is usually supposed to be the same for all cases). Another difficulty may be coinciding ranges, i. e., when the two objects display the characteristic to the same degree. In this case, fractional ranges are used.

The following zero hypothesis is usually formulated:

$$H_0 : \begin{cases} \text{a) random variables} \{x^k\}_{k=\overline{0,p}} \text{ are statistically independent} \\ \text{b) all elementary outcomes are equiprobable, } p = \dfrac{1}{n!} \end{cases} \tag{D.1}$$

There are two popular numerical measures of rank correlation, Spearman coefficient and Kendall coefficient. The former was proposed in 1904 by C. Spearman:

$$\hat{\tau}_{kj}^{(S)} = 1 - \frac{6}{n^3 - n} \sum_{i=1} n \left(x_i^{(k)} - x_i^{(j)} \right)^2 \tag{D.2}$$

If the two rankings coincide, $\hat{\tau}_{kj}^{(S)} = 1$, and when they are opposite, $\hat{\tau}_{kj}^{(S)} = -1$. In all other cases, $|\hat{\tau}_{kj}^{(S)}| < 1$. The formula (D.2) may be generalized to the case of coinciding ranges.

[1] The description follows [11]

The other measure of rank correlation is Kendall coefficient:

$$\hat{\tau}^{(K)}_{kj} = 1 - \frac{4\nu(X^{(k)},\, X^{(j)})}{n(n-1)} \tag{D.3}$$

where $\nu(X^{(k)},\, X^{(j)})$ is the number of neighbor trades of the $X^{(j)}$ sequence necessary to obtain $X^{(k)}$ sequence, or, equivalently, the number of inversions i. e., pairs of sequences elements placed in different order. Again, if the two rankings coincide, $\hat{\tau}^{(K)}_{kj} = 1$, and when they are opposite, $\hat{\tau}^{(K)}_{kj} = -1$; in all other cases, $|\hat{\tau}^{(K)}_{kj}| < 1$.

Determining Kendall coefficient is more computationally intensive than that of Spearman index, but the statistical properties of the former are developed to a greater extent. Moreover, it is more convenient as new observations are added to the sample.

Both of the coefficients may be generalized in the following way. For any bivariate system of n observations

$$\begin{pmatrix} X^{(k)T} \\ X^{(j)T} \end{pmatrix} = \begin{pmatrix} x^{(k)}_1, \dots, x^{(k)}_n \\ x^{(j)}_1, \dots, x^{(j)}_n \end{pmatrix} \tag{D.4}$$

define a rule mapping each pair $(x^{(l)}_{i_1}, x^{(l)}_{i_2})$ a number ("label") $a^{(l)}_{i_1 i_2}$, this rule being negatively symmetric ($a^{(l)}_{i_1 i_2} = -a^{(l)}_{i_2 i_1}$) and centric ($a^{(l)}_{ii} = 0\ \forall l = k, j\ \forall i = 1, \dots, n$). Then a generalized rank correlation coefficient $r^{(g)}$ may be defined as

$$r^{(g)}_{kj} = \frac{\sum_{i_1=1}^{n} \sum_{i_2=1}^{n} a^{(k)}_{i_1 i_2} a^{(j)}_{i_1 i_2}}{\sqrt{\sum_{i_1=1}^{n} \sum_{i_2=1}^{n} {a^{(k)}_{i_1 i_2}}^2 \cdot \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} {a^{(j)}_{i_1 i_2}}^2}} \tag{D.5}$$

The two previously defined correlation coefficients can be expressed in terms of this generalized correlation coefficient as follows: by putting $a^{(l)}_{i_1 i_2} = x^{(l)}_{i_1} - x^{(l)}_{i_2}$, $l = k, j$, one obtains usual correlation coefficient if $x^{(l)}_i$ is a value of l-th variable in i-th observation, and Spearman correlation coefficient $\hat{\tau}^{(S)}_{kj}$, if $x^{(l)}_i$ is the rank of i-th object in the l-th related ranking; and by putting $a^{(l)}_{i_1 i_2} = \operatorname{sign}(x^{(l)}_{i_2} - x^{(l)}_{i_1})$, one obtains formula for Kendall correlation coefficient (D.3). Thus the two coefficients are closely related though Spearman coefficient gives greater weights for more distant pairs, for $n \geq 10$ and $\hat{\tau}^{(S)}_{kj}$, $\hat{\tau}^{(K)}_{kj}$ not close to 1 the approximation being

$$\hat{\tau}^{(S)}_{kj} \approx 1,5\, \hat{\tau}^{(K)}_{kj}$$

.

Testing hypothesis of correlation significance is carried out for relatively large samples ($n \geq 10$) for given significance level α via inequalities

$$|\hat{\tau}^{(S)}| > t_{\alpha/2}(n-2)\sqrt{\frac{1-(\hat{\tau}^{(S)})^2}{n-2}} \tag{D.6}$$

$$|\hat{\tau}^{(K)}| > u_{\alpha/2}\sqrt{\frac{2(2n+5)}{9n(n-1)}} \tag{D.7}$$

where $t_q(\nu)$ and u_q is a 100%q percentile points of Student with ν d. f., and normal distributions. If the inequalities (D.6) and (D.7) hold, H_0 needs to be rejected.

E. Pareto classification

The idea of Pareto classification extends the Pareto relation, or dominance, determined for the two multidimensional objects, to the case of $m > 2$ objects. This approach was proposed by V. V. Shakin [21].

The p-th n-dimensional object, or observation, $x^{(p)} = (x_1^{(p)}, \dots, x_n^{(p)})$ is said to *weakly Pareto-dominate* q-th object $x^{(q)}$, $p \neq q$, $q \in \{1, \dots, m\}$ (denote this as $x^{(p)} \succeq x^{(q)}$) if for all $k = 1, \dots, n$, $x_k^{(p)} \geq x_k^{(q)}$. Another popular notion is that $x^{(p)}$ is a *weak Pareto improvement* of $x^{(q)}$. Pareto dominance (improvement) is called *strict* (or simply *Pareto dominance*) if, besides, there exists $l \in \{1, \dots, n\}$ such that $x_l^{(p)} > x_l^{(q)}$. Then we denote this as $x^{(p)} \succ x^{(q)}$.

The algorithm of Pareto classification determines consequently Pareto boundaries of the superior (majorant) objects, as compared to all the others. The first layer consists of observations $X^{(1)} = \{x^{(p)} | p \in I_1\}$, $I_1 \subset \{1, \dots, n\}$ such that there are no objects "better", in Pareto sense, i. e., in all respects, than $x^{(p)}$ from the first Pareto layer $X^{(1)}$: $\forall\, q \neq p,\ q = 1, \dots, m\ x^{(q)} \not\succeq x^{(p)}$. This is a set of Pareto optimal points: for neither of them the situation can be improved in any factors without making some other factors worse. When all such objects are determined, the process recursively continues for the sample with elements from $X^{(1)}$ thrown out until all objects are given their respective class numbers: $x^{(p)} \in X^{(2)} \Leftrightarrow \forall\, x^{(q)} \succ x^{(p)}\ x^{(q)} \in X^{(1)}$, $x^{(p)} \in X^{(3)} \Leftrightarrow \forall\, x^{(q)} \succ x^{(p)}\ x^{(q)} \in X^{(1)} \cup X^{(2)}$, etc.

If for all p the components of the n-dimensional vector $x^{(p)}$ are perfectly correlated (or if the objects are characterized by the only one dimension), this procedure will find m classes $I_1, \dots, I_m$. On the other hand, if the dimensionality of vectors being compared is too large (greater than the very number of objects), and the components are not correlated (or only weakly correlated), it is likely that all objects would belong to the same (first) Pareto class.

The process of classification may go in the other direction, namely, selecting the "worst" objects first. This procedure will result in a different classification, with different allocation of objects among classes and, possibly, different number of classes. These two ways can be reciprocated by simply negating the components of vectors $x^{(p)}$. It can be directly shown that objects with extreme realizations of the factors ($\exists\, k,\, l \mid x_k^{(p)} = 1,\ x_l^{(p)} = 0$ in our normalization) will fall into the first class in both classifications.

The main advantages of this approach as compared to the regression and principal components methods described above is its non-compensatory property. Besides, this method is scale-independent; in fact, it suffices to provide rankings of the objects to build Pareto classification.

The non-compensatory principle, as described in Chapter I, postulates impossibility to compensate deterioration in one of the indicators, or components, by improvements in others. In fact, this "transfer" makes the two states of Nature (the initial one and the perturbed one) Pareto uncomparable: neither of them will constitute Pareto improvement over the other.

F. Rankings of Regions of Russia

The principal components analysis (see Section III.3) and Pareto-classification (see Section III.4) of data for the regions of Russian Federation were carried out. There were two sets of variables used in principal components indentification: a broader one with 14 variables, and a narrower one with 7 variables similar to those used in international quality of life comparisons. The corresponding rankings are given in columns 2 and 3 of the table below. For Pareto rankings, the same set of 14 variables was used (column 4) as well as a set of factors selected by one from each of the main groups outlined in Introduction: GRP per capita, life expectacny at birth, population per doctor, dwelling per capita, and emissions.

Region	Principal components		Pareto ranks	
	All vars	Selected vars	All vars	Selected vars
Northern Regions				
Resp. Karelia	12	12	1	2
Resp. Komi	4	2	1	2
Arkhangelskaya obl.	18	15	1	2
Vologdskaya obl.	17	13	1	1
Murmanskaya obl.	14	9	1	2
North-Western Regions				
St. Peterburg, city	32	30	1	1
Leningradskaya obl.	53	37	1	1
Novgorodskaya obl.	34	34	1	2
Pskovskaya obl.	59	52	1	1

Region	Principal componetns		Pareto ranks	
	All vars	Selected vars	All vars	Selected vars
Central Regions				
Bryanskaya obl.	71	65	1	2
Vladimirskaya obl.	70	66	1	2
Ivanovskaya obl.	37	50	1	2
Kaluzhskaya obl.	57	59	1	2
Kostromskaya obl.	50	48	1	2
Moscow, city	11	23	1	1
Moscovskaya obl.	69	73	1	2
Orlovskaya obl.	62	63	1	2
Ryazanskaya obl.	20	33	1	1
Smolenskaya obl.	35	45	1	1
Tverskaya obl.	38	41	1	1
Tul'skaya obl.	29	38	1	3
Yaroslavskaya obl.	31	32	1	1
Volga–Vyatka Regions				
Resp. Mary El	68	56	1	3
Resp. Mordovia	54	60	1	1
Resp. Chuvashia	60	64	1	2
Kirovskaya obl.	41	44	1	3
Nizhegorodskaya obl.	47	47	1	2
Central Chernozemye Regions				
Belgorodskaya obl.	61	67	1	1
Voronezhskaya obl.	63	71	1	1
Kurskaya obl.	55	62	1	1
Lipetskaya obl.	26	35	1	1
Tambovskaya obl.	73	70	1	2

Region	Principal componetns All vars	Principal componetns Selected vars	Pareto ranks All vars	Pareto ranks Selected vars
Povolzhye Regions				
Resp. Kalmykia	64	61	1	2
Resp. Tatarstan	39	51	1	1
Astrakhanskaya obl.	44	49	1	2
Volgogradskaya obl.	58	58	1	1
Penzeskaya obl.	75	72	1	1
Samarskaya obl.	24	28	1	1
Saratovskaya obl.	48	54	1	1
Ulyanovskaya obl.	51	53	1	2
Northern Caucasus Regions				
Resp. Adygeya	72	74	1	1
Resp. Dagestan	78	78	1	1
Resp. Ingushetia	79	79	1	1
Kabardino-Balkarskaya Resp.	76	76	1	1
Karachaevo-Cherkesskaya. Resp.	77	77	1	1
Resp. Alania	74	75	1	1
Krasnodarsky kray	66	68	1	2
Stavropolsky kray	67	69	1	1
Rostovskaya obl.	65	55	1	2
Ural Regions				
Resp. Bashkortostan	43	46	1	2
Udmurtia Resp.	23	27	1	3
Kurganskaya obl.	56	36	2	3
Orenburgskaya obl.	16	20	1	2
Permskaya obl.	15	19	1	2
Sverdlovskaya obl.	19	17	1	2

Region	Principal componetns		Pareto ranks	
	All vars	Selected vars	All vars	Selected vars
Chelyabinskaya obl.	22	21	1	2
Western Siberia Regions				
Resp. Altay	36	39	1	4
Altaysky kray	49	42	1	3
Kemerovskaya obl.	6	11	1	2
Novosibirskaya obl.	46	29	1	2
Omskaya obl.	27	24	1	1
Tomskaya obl.	28	18	2	2
Tyumenskaya obl.	3	3	1	1
Eastern Siberia Regions				
Resp. Buryatia	45	40	1	4
Resp. Tyva	1	6	1	5
Resp. Hakasia	25	26	1	3
Krasnoyarsky kray	5	5	1	2
Irkutskaya obl.	10	14	1	2
Chitinskaya obl.	30	25	1	4
Far-Eastern Regions				
Resp. Saha (Yakutia)	9	10	1	1
Evreyskaya a.o.	40	31	1	4
Chukotsky a.o.	2	1	1	1
Primorsky kray	33	22	2	2
Khabarovsky kray	21	16	2	2
Amurskaya obl.	42	43	1	3
Kamchatskaya obl.	13	8	1	2
Magadanskaya obl.	7	4	1	1
Sakhalinskaya obl.	8	7	1	2
Kaliningradskaya obl.	52	57	1	4

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